# Back-in-time dynamics of the cluster IE 0657-56 (the Bullet System)

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#### ABSTRACT

We present a simplified dynamical model of the "Bullet" system of two colliding clusters. The model constrains the masses of the system by requiring that the orbits of the main and sub components satisfy the cosmological initial conditions of vanishing physical separation a Hubble time ago. This is also known as the timing argument. The model considers a system embedded in an over-dense region. We argue that a relative speed of 4500 km/s between the two components is consistent with cosmological conditions if the system is of a total mass of  $2.8 \times 10^{15} h^{-1} M_{\odot}$  is embedded in a region of a (mild) over-density of 10 times the cosmological background density. Combining this with the lensing measurements of the projected mass, the model yields a ratio of 3:1 for the mass of the main relative to that of the subcomponent. The effect of the background weakens as the relative speed between the two components is decreased. For relative speeds lower than  $\sim 3700 \text{km/s}$ , the timing argument yields masses which are too low to be consistent with lensing.

**Key words:** cosmology: theory, large-scale structure of the Universe — clusters

# 1 INTRODUCTION

The system IE 0657-56 of two colliding clusters (termed the "Bullet"), is particularly interesting among such systems. X-ray observations of this system show a prominent bow-shock generated by the supersonic motion (Mach number of  $\mathcal{M}=3\pm0.4$ ) of the sub-cluster (the Bullet) relative to the gaseous component of the main cluster. The estimated speed of the Bullet relative to the main cluster is  $\sim 4700 \mathrm{km/s}$ , while the relative line-of-sight velocities between the groups of galaxies associated with the two components is only 600km/s. Therefore, the relative motion between the two clusters is almost entirely perpendicular to the line-of-sight (Barrena et al. 2002; Markevitch et al. 2004).

The current dynamical state of the system must be consistent with that evolved in an expanding Universe. Therefore, all matter belonging to the system must originate from a region of vanishing size as we approach the initial singularity  $(t \to 0)$ . The positions of objects in the system can be uniquely moved back-in-time from their current positions and velocities. The masses of these objects can then be tuned so that the cosmological constraint at  $t \to 0$  is satisfied. For a two component system, this is traditionally known as the

timing argument and it has been used for studying the system of the Galaxy and M31 (e.g. Kahn & Woltjer 19969; Peebles 1993). Timing arguments have also been applied for the the Bullet system by Zhao (2007). This study has not considered systems embedded in an over-dense region of a larger scale. Further, it differs in the details from the model presented here.

The outline of this paper is as follows. In  $\S 2$  we briefly present the lensing measurements and constrain the masses of the main and sub components assuming NFW forms (see below) for their density profiles. In  $\S 3$  we describe our model and present the results for the mass estimates. We conclude in  $\S 4$  with a general discussion.

## 2 LENSING CONSTRAINTS

We assume that the density profiles of dark matter halos of the main and the sub components follow the form given by Navarro, Frenk & White (1996) (hereafter, NFW) as

$$\rho(r) = \frac{\rho_{\text{crit}}\delta_0}{x(1+cx)^2} \,, \tag{1}$$

where c is the concentration parameter,  $x = r/R_v$  is the distance from the halo center in units of the virial radius, and  $\rho_{\rm crit} = 3H^2/(8\pi G)$  is the critical mean density  $(H = \dot{a}/a)$ where a is the scale factor). Here we assume that the mean density within the virial radius is  $178\rho_{\rm crit}$ . This gives  $\delta_0$  to be  $\delta_0 = (178/3)c^2/[\ln(1+c) - x/(1+c)]$ 

The profile (1) depends on the halo concentration, c, and its virial radius,  $R_{\rm v}$ . We determine these two parameters for each halo by matching the combined weak and strong lensing estimates of the projected mass density as given in figure (5) of Bradač et el. (2006). For convenience, we borrow the results for the projected mass in that figure and show them in figure (1) here. Points extending to 0.5 and 0.3Mpc correspond to the main and sub components, respectively. Attached to the points are  $1\sigma$  error-bars, taken as 7% and 9% of the mass for the main and the sub-cluster, respectively (Bradač et el. 2006). We fit NFW profiles to these data by minimizing the quantity

$$\tilde{\chi} = \sum_{\alpha} \sigma^{-1}{}_{\alpha} |M_{\alpha}^{\text{lens}} - M_{\alpha}^{\text{NFW}}| \tag{2}$$

with respect to  $R_{\rm v}$  and c. Here the summation is over all data points,  $M^{\rm lens}$  and  $M^{\rm NFW}$  are projected masses taken from the lensing measurements and the NFW profile, and  $\sigma_{\alpha}$  are  $1\sigma$  error-bars at each data point. This procedure yields multiple solutions for  $R_{\rm v}$  and c for both dark components. The results are represented in figure (2) as contour plot of the quantity  $\tilde{\chi}$  as a function of  $R_{\rm v}$  and c for the main (top panel) and sub (bottom) components. There is a clear degeneracy between  $R_{\rm v}$  and c. The difference between two solutions with similar  $\tilde{\chi}$  is illustrated in figure (1) where, in addition to the data, we show the projected mass corresponding to NFW profiles with  $R_{\rm v} = 3.2 \& 2.6 \,{\rm Mpc}$  (for the main) and  $R_{\rm v}=2.9~\&~2.3~{\rm Mpc}$  (for the sub), as indicated in the figure. These values of  $R_{\rm v}$ , respectively for the main and the sub, represent the lower and upper limits of what we consider as acceptable match to the mensing measurements. Therefore, the largest acceptable ratio between the virial radii of the main and sub components is  $3.2\,:\,2.3\,=\,1.4\,:\,1.$  The mass of a halo within a sphere of radius  $R_{\rm v}$  is  $M_{\rm v}=10^{14}\left(\frac{h}{0.7}\right)^2\left(\frac{R_{\rm v}}{1{\rm Mpc}}\right)^3M_{\odot}$ . Therefore, the largest mass ratio allowed by these considerations is  $1.4^3:1\approx 3:1$ . We will see in the next section that this ration is also consistent with cosmological initial conditions if the Bullet system resides in a region of a mild over-density of 10 times the background density.

#### MAIN-SUB DYNAMICS

We give here the equations governing the evolution of the two dark components of the system IE 0657-56. We neglect the gravity of gas in the system. We consider NFW profiles truncated at the virial radii for both components. The system is assumed to be embedded in region of density  $(1 + \delta(t))\bar{\rho}(t)$  where  $\bar{\rho}$  is the mean cosmic matter density and  $\delta$  is the density contrast with a time dependence derived from the spherical top-hat model. We assume a flat universe with a cosmological constant  $\rho_{\rm v}(t) = const$  to the density

from the dark energy. The values of the density parameters are  $\rho_{\rm v}/\rho_{\rm crit}=0.7$  and  $\bar{\rho}/\rho_{\rm crit}=0.3$ . We work in the system of the center of mass of the two clusters. The motion of the center of mass,  $\mathbf{R}_{i}$ , of the component i is determined by

$$\ddot{\mathbf{R}}_{i} = \frac{\mathbf{F}_{i,j}}{M_{i}} - \frac{4\pi}{3}G\left[\bar{\rho}(1+\delta) - 2\rho_{v}\right]\mathbf{R}_{i}$$
(3)

where  $M_i$  is the mass of each component. For a given value of  $\delta$  today, the function  $\delta(t)$  is obtained from the spherical collapse model. The force,  $\mathbf{F}_{i,j}$ , the halo j applies to i, is computed for NFW profiles for both components.

For the mass,  $M_{\rm m}$ , of the main component we consider the range  $(1.5-3.4)\times 10^{15}M_{\odot}10^{15}$ . This corresponds to  $R_{\rm v}=(2.46-3.23){\rm Mpc}$ . According to figure (2),  $R_{\rm v}=$ 2.46Mpc is ruled out by lensing for all values of the concentration parameter, but we explore it anyways and show that such masses are not favored by the timing argument as well. For the mass,  $M_{\rm s}$  of the sub, we consider the range  $M_{\rm s}=$  $(0.34-2.8) \times 10^{15} M_{\odot}$ , corresponding to  $R_{\rm v} = (1.5-3) {\rm Mpc}$ . The lower end of this range is inconsistent with lensing. The concentration parameters, c, are chosen to be those giving the best match to the lensing data (see fig. 2).

Given a choice of  $M_{\rm m}$  and  $M_{\rm s}$  we numerically solve the equations (3) back-in-time for three values of the current relative velocity, V:4500,4000 and 3700km/s. We assume a distance of 0.7 Mpc between the mass centers of the two components. For simplicity we assume that the relative speed lies along the separation between the centers of mass of the cluster components.

Figure (3) shows the separation,  $R_{\rm s} - R_{\rm m}$ , between the two components at  $t \to 0$ , as contour plots in the plane of  $M_{\rm m}$  and  $M_{\rm s}$ . The top, middle and bottom rows, respectively, V = 4500, 4000 and 3700km/s. The column to the left shows solutions obtained for a cluster embedded in mean cosmological density, while the one to the right, to a nonlinear density contrast of 10 at the measured redshift of the system. Each panel in the middle and bottom rows contains more than one set of contours. The set corresponding to the smallest masses represent the relevant solutions in which the separation reaches a maximum value only once within a Hubble time. The separations at t=0 are sensitive to the assumed relative speed, V. For example, according to the left panels, for  $M_{\rm m}=2\times10^{15}M_{\odot}$  and  $M_{\rm s}=10^{15}M_{\odot}$ , the separation is zero for V = 4000 km/s, while it is  $\sim 16 \text{Mpc}$ for V = 4500 km/s. For V = 4500 km/s

We have seen in the previous section that the lensing analysis gives 3:1 as the largest ratio between the mass of the main and the sub-components. Inspection of figure (3) reveals that this is also consistent with the requirement of cosmological initial conditions if the system is embedded in a region which 10 times denser than the background (topright panel). The inferred total mass from this figure is  $4 \times$  $10^{15} M_{\odot} = 2.8 h^{-1} 10^{15} M_{\odot}$  which is close to the mass inferred from lensing for a 3:1 mass ratio.

The effect of having an over-dense region engulfing the system is most pronounced for the largest relative speed (cf the three rows to the right) and is almost unnoticeable for the smallest speed. The reason is as follows. The existence of this region has little effect on the dynamics when the cluster

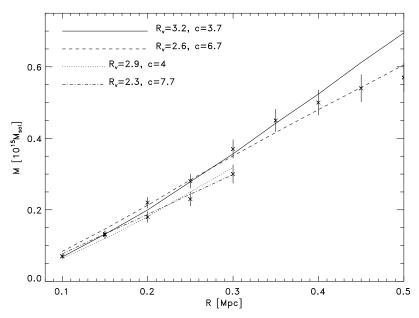


Figure 1. The projected masses of the colliding clusters, as estimated lensing analyses of Bradač et al. (2006). The upper points extending to 0.5Mpc show the main halo, while the lower correspond to the sub halo. The lines show NFW projected mass profiles.

components are still close to each other. As we go sufficiently backward in time, their separation increases and so their mutual gravity decreases. As this happens, the density of the region decreases, in accordance with the top-hat model. For large relative velocities, large separations are reached while the surrounding region is still at a significant over-density and thus can affect the dynamics.

# 4 DISCUSSION

The relative velocity between the two cluster components is large compared to the usual virial speeds ( $\sim 10^3 \text{km/s}$ ). This triggered discussions on whether or not the system if consistent with the currently viable models for structure formation based on the Cold Dark Matter Scenario (CDM) with a cosmological constant,  $\Lambda$ , i.e. the  $\Lambda$ CDM model. Modified Newtonian Dynamics (MOND) has been invoked (Angus, Famaey, & Zhao 2006, Angus & McGaugh 2007) to explain the estimated relative speed. Other explanations relied on long range scalar interactions in the dark sector to aid gravity in boosting the relative speed (Farrar & Rosen 2007). Nonetheless, like in the Newtonian case both of these suggestions need to be assessed in a cosmological context. Scalar interactions modify large scale structure formation in a desirable way (Nusser, Gubser & Peebles 2005; Farrar & Peebles 2004). Still, proper cosmological simulations need to be performed to assess whether or not they could easily reproduce a "Bullet system". As for MOND-like modifications (Bekenstein 2004), it is still unclear how they can be implemented in a cosmological context. The simplest cosmological adaptation had been incorporated in a cosmological simulation and lead to large scale structure formation similar to ΛCDM, but with MOND's acceleration parameter which is smaller by an order of magnitude that the standard value (Nusser 2000).

We argue here that even with a large value for the relative speed, gravitational force alone is consistent with cosmological initial conditions, i.e. the timing argument. The total mass we derive is  $2.8h^{-1}10^{15}M_{\odot}$ , if the system resides in a region of a density contrast of about 10. Hayashi & White (2006) performed a likelihood analysis on the Millennium simulation to conclude that the Bullet system can be accommodated in the LCDM cosmogony. They adopt masses of  $2.16 \times 10^{15} h^{-1} M_{\odot}$  and  $5.3 \times 10^{13} h^{-1} M_{\odot}$  for the main and sub components, respectively. Our mass estimate is slightly larger than theirs but we still could be consistent with the  $\Lambda$ CDM model normalized to  $\sigma_8 = 0.9$ . However, the mass ratio we advocate here, 3:1, is quite different from the value adopted by Hayashi & White (2006). Zhao (2007) also used the timing argument to constrain the mass of the Bullet system without taking into account dynamical effects of an over-dense surrounding environment. The mutual force in that study is also computed differently from this work. Zhao's estimate of the total mass is as twice as the value derived here.

Springel & Farrar (2007) used non-cosmological simulations of two colliding clusters of a mass ratio of 10:1 and found that the relative speed between the two dark components could be as small as 2700 km/s, while the shock speed is 4500 km/s. As mentioned before, a 10:1 mass ratio is inconsistent with lensing. Of course, the physical effects leading to velocity lag of dark mass relative to the shock could still be important and they need to be quantified with simulations having a mass ration of 3:1. Nonetheless, for low relative speeds the timing argument yields masses (see bottom-left panel in fig. 3) which are too small to be consistent with the lensing measurements.

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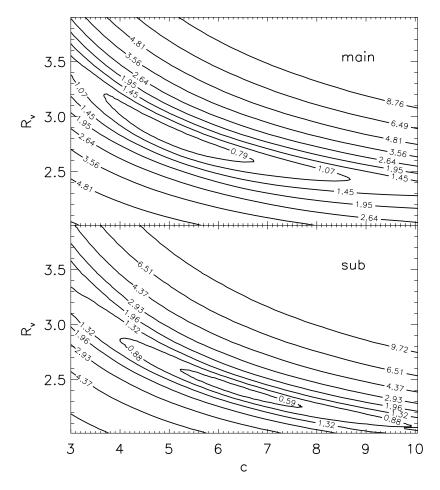


Figure 2. Contours of the quantity  $\tilde{\chi}$  which represents the goodness-of-fit of NFW profiles to the lensing data, in the plane of the concentration parameter c and and the virial radius  $R_{\rm v}$  in units of Mpc. (the virial mass is  $M_{\rm v}=10^{14}(R_{\rm v}/{\rm Mpc})^3M_{\odot}$ ). Contour levels are indicated by the numbers attached to the curves.

We have made some attempts, the details of which are not presented here, at including dynamical friction and tidal stripping according to the recipes in Nusser & Sheth (1999). These effects reduce the relative speed between the two components, therefore, for a given final relative speed they increase corresponding masses required to match the timing constraint.

The analysis presented here neglects important processes which might affect the dynamics of the system. Each of the two components have formed by merging of smaller halos. Therefore, sometime in the past, the matter making each component was distributed in more than one clump. At that time, the mutual force determining the relative motion of the centers of mass of the two components should depend on the spatial distribution of their progenitors. If, however, major merging and accretion activities in both components ceased when the separation between them was large enough. Then the monopole term alone, which we use here for the mutual force, should be a reasonable description to the dynamics of the system.

We have tuned the masses so that the solution back-intime gives vanishing separation near t=0. This has been done by solving the initial value problem of given current separation and relative speed as input to the numerical solution. Alternatively, we could have solved a boundary value problem where the first boundary condition is current separation and the second is a constraint which guarantees the cosmological constraint near t=0 (e.g. Peebles 1993, Nusser & Branchini). This could be done using the least action principle, as proposed by Peebles (1989). This approach yields a prediction for the current relative velocity for a given mass choice. This predicted velocity could then be compared with the observed speed. This approach will be employed in future work.

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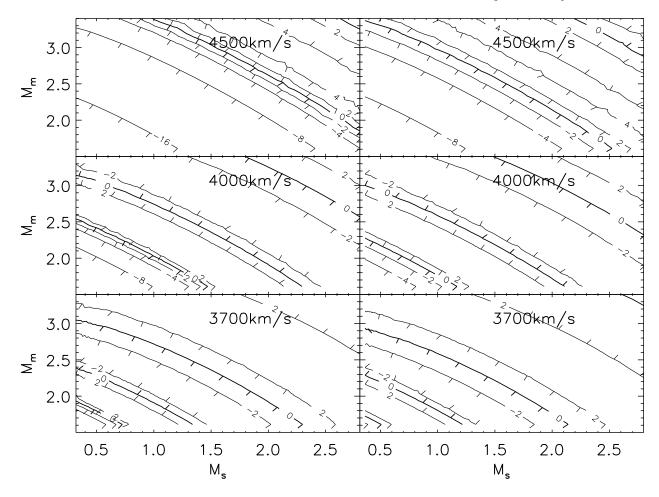


Figure 3. Contours of the separation,  $R_{\rm s}-R_{\rm m}$ , between the centers of mass of the main and sub components (in units of  $10^{15} M_{\odot}$ ) at  $t\to 0$ , as obtained from back-in-time solutions of the equations of motion. The contours are plotted in the plane of the assumed masses of the two components. The contour levels correspond to (physical) separations of -16,-8,-4,-2,0,2,4 and 6 Mpc, as indicated on some of the contours. Thick lines indicate zero separations and ticks indicate downhill directions. In each panel in the middle and bottom rows, the relevant set of contours are those lying nearer to the lower masses. For this set, the centers of mass of the two components originate from the same point and reach a maximum separation only once within a Hubble time.

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